Solving linear systems by elimination with multiplication and addition

Goal Find the same pair (x, y) that makes BOTH linear equations true. Here x and y are unknown numbers.

From Topic 28 You multiplied by −1. Now choose multipliers that make opposite terms. Then add as before.

A Choose and prepare

Start with these two original equations:

$$\begin{aligned} -2x + 7y &= -3 \\ 4x + 5y &= 25 \end{aligned}$$

We want the x terms to cancel.

The terms −2x and +4x will not cancel. We need −4x and +4x. Two copies of −2x make −4x, so choose ×2 for the first equation.

Why we can multiply by 2

These are two copies of the FIRST equation:

$$\begin{array}{rrcr} & -2x & +7y & = & -3 \\ + & -2x & +7y & = & -3 \\ \hline & -4x & +14y & = & -6 \end{array}$$

Equal sides give equal totals. This doubles EVERY term on BOTH sides:

$$2(-2x + 7y) = 2(-3)$$

We can divide both sides by 2 to get the original back. The same pairs still work.

The system is ready to add

First equation doubled; second unchanged:

$$\begin{aligned} -4x + 14y &= -6 \\ 4x + 5y &= 25 \end{aligned}$$

Next column: Section A, "Add and finish."

A Add and finish

Add BOTH left sides and BOTH right sides.

$$\begin{array}{rrcr} & -4x & +14y & = & -6 \\ + & 4x & +5y & = & 25 \\ \hline & 0 & +19y & = & 19 \end{array}$$

Only y remains. Divide BOTH sides by 19:

$$\begin{aligned} \frac{19y}{19} &= \frac{19}{19} \\ y &= 1 \end{aligned}$$

One value found. Now find the other.

We know y = 1. Replace y with 1 in either original. Here we use the second:

$$\begin{aligned} 4x + 5y &= 25 \\ 4x + 5(1) &= 25 \end{aligned}$$

Subtract 5, then divide by 4, on BOTH sides:

$$\begin{aligned} 4x + 5 - 5 &= 25 - 5 \\ 4x &= 20 \\ \frac{4x}{4} &= \frac{20}{4} \\ x &= 5 \end{aligned}$$

Solution: (5, 1). Write x first, y second.

Put x = 5 and y = 1 into BOTH originals. Each must be true:

$$\begin{aligned} -2(5) + 7(1) &= -3 \\ -3 &= -3 \\ 4(5) + 5(1) &= 25 \\ 25 &= 25 \end{aligned}$$

B New system: prepare both

$$\begin{aligned} -3x + 7y &= -16 \\ 5x - 6y &= 21 \end{aligned}$$

3 and 5 both reach 15. First ×5 gives −15x; second ×3 gives +15x. Multiply BOTH sides:

$$\begin{aligned} 5(-3x + 7y) &= 5(-16) \\ 3(5x - 6y) &= 3(21) \end{aligned}$$

Now add. The opposite x terms cancel:

$$\begin{array}{rrcr} & -15x & +35y & = & -80 \\ + & 15x & -18y & = & 63 \\ \hline & 0 & +17y & = & -17 \end{array}$$

Only y remains. Divide BOTH sides by 17:

$$\begin{aligned} \frac{17y}{17} &= \frac{-17}{17} \\ y &= -1 \end{aligned}$$

We know y = −1. Use the first original to find x. Add 7, then divide by −3, on BOTH sides:

$$\begin{aligned} -3x + 7(-1) &= -16 \\ -3x - 7 + 7 &= -16 + 7 \\ -3x &= -9 \\ \frac{-3x}{-3} &= \frac{-9}{-3} \\ x &= 3 \end{aligned}$$

Solution: (3, −1). Check BOTH originals.

$$\begin{aligned} -3(3) + 7(-1) &= -16 \\ -16 &= -16 \\ 5(3) - 6(-1) &= 21 \\ 21 &= 21 \end{aligned}$$

TOPIC ID: **29-C3**	TOPIC TITLE: Linear systems by multiplying and adding	FORESTHILL HIGH SCHOOL
NAME:	BLOCK:	DATE:

LEARNING PAPER EVIDENCE

Solve and show how you know

Use the printed equations. Work vertically; show changes to BOTH sides. Find (x, y) and check BOTH originals.
You may eliminate either variable. A zero value is allowed; it does not mean there is no solution.

(a) Prepare with a negative multiplier

Original equations:

$$-5x + 6y = 18$$
$$-3x + 2y = 6$$

We need −6y to cancel +6y. Since (−3)(2y) = −6y, multiply the WHOLE second equation by −3:

$$(-3)(-3x) + (-3)(2y) = (-3)(6)$$

Ready to add: first unchanged; second ×(−3).

$$-5x + 6y = 18$$
$$9x - 6y = -18$$

Your turn Add and find both values

Solution (x, y): (_____ , _____)

Check both originals

(b) Choose both multipliers

$$8x - 5y = -22$$
$$9x + 2y = 21$$

Hint: make −5y and +2y opposites. Choose both multipliers.

Find both values

Solution (x, y): (_____ , _____)

Check both originals

(c) Make your own plan

$$-9x - 4y = -14$$
$$-5x - 3y = -7$$

Solve independently. Show your method and checks.

Find both values

Solution (x, y): (_____ , _____)

Check both originals